CONCURSUL NAȚIONAL DE MATEMATICĂ APLICATĂ „ADOLF HAIMOVICI”
Barem de corectare
clasa a IX – a
Filiera tehnologică – Profil tehnic – toate specializările

1)

a) Fie A mulţimea elevilor care au participat la Haimovici şi B mulţimea elevilor care au participat la concursul de română.

$card(A \cup B) = 30 - 5 = 25 \Rightarrow cardA + cardB - card(A \cap B) = 25 \Rightarrow card(A \cap B) = 14$

Deci 14 elevi au participat la ambele concursuri. **1p**

16-14=2 elevi au participat numai la Haimovici **0,5p**

23-14=9 elevi au participat numai la concursul de limba şi literatura română **0,5p**

b) $\begin{cases} y^2 = xz \\ x+y+z=114 \\ y+36=\dfrac{x+z}{2} \end{cases} \Leftrightarrow \begin{cases} y^2 = xz \\ x+y+z=114 \\ x+z=2y+72 \end{cases} \Leftrightarrow \begin{cases} y^2 = xz \\ 3y=42 \\ y+36=\dfrac{x+z}{2} \end{cases} \Leftrightarrow \begin{cases} xz=196 \\ y=14 \\ x+z=100 \end{cases}.$ **1p**

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{x+z}{xz}+\frac{1}{14}=\frac{100}{196}+\frac{1}{14}=\frac{50}{98}+\frac{7}{98}=\frac{57}{98}$ **1p**

c) $3,\ 7,\ 11, \dots, x$ sunt termenii unei progresii aritmetice cu $a_1 = 3,\ r = 4,\ S_n = 465.$ **1 p**

Nu cunoaştem $n.$ $x = a_n = a_1 + (n-1)r = 3 + (n-1)4 = 4n - 1.$

$\frac{n(4n+2)}{2} = 465.$ **1 p**

Determinarea lui $n \in \mathbf{N}^*, n = 15$ **0,5p**

$x = a_{15} = a_1 + 14r = 59.$ **0,5p**

2)

a) Metoda inducţiei matematice. 1° Etapa de verificare $P(1)$ (A) **1 p**

2° Fie $k \geq 1. P.p. P(k)(A).$ Demonstrația $P(k) \rightarrow P(k+1)$ **1,5 p**

Din 1° şi 2° $\Rightarrow P(n)(A)$ **0,5 p**

b) $\sqrt{\frac{a}{bcd}} = \sqrt{\frac{a^2}{abcd}} = \frac{a}{\sqrt{abcd}}$ deci progresia este $\frac{a}{\sqrt{abcd}} = \frac{b}{\sqrt{abcd}} = \frac{c}{\sqrt{abcd}} = \frac{d}{\sqrt{abcd}}.$ **1 p**

$\frac{b}{\sqrt{abcd}} = \frac{a}{\sqrt{abcd}} + r \Rightarrow b = a + r\sqrt{abcd}$ **1 p**

La fel $c = b + r\sqrt{abcd}$ și $d = c + r\sqrt{abcd}$. Deci a, b, c, d formează o progresie aritmetică cu raţia $r\sqrt{abcd}$, unde r este raţia progresiei iniţiale. **2 p**

3) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AO}$, $\overrightarrow{AD} = 2 \cdot \overrightarrow{AO}$, $\overrightarrow{AE} = \overrightarrow{AF} + \overrightarrow{AO}$, $\overrightarrow{AB} + \overrightarrow{AF} = \overrightarrow{AO}$ **2p**

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\overrightarrow{AB}+\left(\overrightarrow{AB}+\overrightarrow{AO}\right)+2\overrightarrow{AO}+\left(\overrightarrow{AF}+\overrightarrow{AO}\right)+\overrightarrow{AF}=$$
$$=4\overrightarrow{AO}+2\left(\overrightarrow{AB}+\overrightarrow{AF}\right)=4\overrightarrow{AO}+2\overrightarrow{AO}=6\overrightarrow{AO}$$ **1p**

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=x\cdot\overrightarrow{AO}\Leftrightarrow x\cdot\overrightarrow{AO}=6\cdot\overrightarrow{AO}\Leftrightarrow x=6$$ **1p**

$$\overrightarrow{BD}+\overrightarrow{BF}=\left(\overrightarrow{BC}+\overrightarrow{BO}\right)+\left(\overrightarrow{BO}+\overrightarrow{BA}\right)=2\overrightarrow{BO}+\left(\overrightarrow{BC}+\overrightarrow{BA}\right)=2\overrightarrow{BO}+\overrightarrow{BO}=3\overrightarrow{BO}$$ **2p**

$$\overrightarrow{BD}+\overrightarrow{BF}=y\cdot\overrightarrow{BO}\Leftrightarrow 3\cdot\overrightarrow{BO}=y\cdot\overrightarrow{BO}\Leftrightarrow y=3$$ **1p**

4) a) $a=\sum_{k=1}^{n}\frac{2k+1}{k^2\cdot(k+1)^2}=\sum_{k=1}^{n}\frac{(k+1)^2-k^2}{k^2\cdot(k+1)^2}=\sum_{k=1}^{n}\left(\frac{1}{k^2}-\frac{1}{(k+1)^2}\right)=1-\frac{1}{(n+1)^2}$ **2p**

$$a=1-\frac{1}{(n+1)^2}\Rightarrow a\in(0,1),\forall n\in N^*\Rightarrow[a]=0,\{a\}=a-[a]=1-\frac{1}{(n+1)^2}=\frac{n^2+2n}{(n+1)^2}$$ **2p**

b) $\{a\}=0{,}9999\Leftrightarrow 1-\frac{1}{(n+1)^2}=\frac{9999}{10000}\Leftrightarrow 1-\frac{1}{(n+1)^2}=1-\frac{1}{10000}\Leftrightarrow$ **2p**

$$\Leftrightarrow(n+1)^2=10000\overset{n\in N^*}{\Leftrightarrow}n+1=100\Leftrightarrow n=99$$ **1p**